EECS 70   Discrete Mathematics and Probability Theory
Fall 2014   Anant Sahai

Homework 1

This homework is due September 8, 2014, at 12:00 noon.

Administrative details of preparing and submitting homework submissions: In EECS 70 this semester, you will be submitting all homework solutions online. (You might recognize the submission system from EECS 61ABC.) Use your instructional account and follow the same approach you were instructed to use for Homework 1. See your GSI or Piazza for instructions on how to do this.

You are welcome to form small groups (up to four people) to work through the homework, but you **must** write up all your solutions on your own and give credit to the people you worked with or anyone who helped you (this will not diminish the credit you receive).

Although your final submission must be neat (and so typesetting with LaTeXis not a bad option), you are strongly encouraged **not** to directly try to solve the problems in any sort of typesetting environment. Use paper and pencil and scratch paper. Only typeset your solutions after you already pretty-much know exactly what you want to write. Otherwise, you risk wasting a lot of time.

Before submitting, make sure you check carefully that the PDF comes out correctly and correct any errors. We suggest using

```
acroread hw1.pdf
```

Your submission needs to start with the following information:

> Your full name
> Your login name
> The name of the homework assignment (e.g. hw1)
> The number of the problem (e.g. 1)
> Your section number
> Your list of partners for this homework, or "none" if you had no partners

To submit your answers to this homework assignment, create a directory named `hw1`, copy your solution file (hw1.pdf) to that directory, `cd` to that directory, and then give the command

```
submit hw1
```

Note that the file you submit must be called `hw1.pdf`.

1. **Virtual Lab**

   As part of the homework, you will be asked to complete "Virtual Labs" that involve programming and plotting things. Knowing how to simulate and explore what actually can happen is an important skill and is very useful in learning the material and developing an intuition for it. To help students in this process, the course staff has prepared an official EECS 70 virtual machine with a recent version of Ubuntu Linux, Python 2.7, and several libraries we'll be using throughout the class.

   For problems that have a significant programming component, we will usually provide IPython Notebook templates that students can use. However, solutions are generally accepted in any programming language that students prefer. The course staff will only support Python/IPython as the official tool to complete the Virtual Labs.

   1. Please set up your Virtual Machine according to the instructions on the course web page, available at

      `http://www-inst.eecs.berkeley.edu/~cs70/fa14/vm/setup.html`

      Once you are done, "answer" this question by writing "I have read the instructions and setup my Virtual Machine accordingly" and signing your name if possible.

   2. Start up the Virtual Machine and enter the password as seen on the screen. Launch a terminal (Ctrl + Alt + t), and change into the "cs70" directory (`cd cs70`).

      Type `ipython notebook`. In the browser window that pops up, click on the "python_sample" notebook. This will open up a new tab, which contains one code cell with a few lines of code. Don't worry if you don't understand the code – it will be explained later.

      Finally, either click on Cell and select Run, or click on the "Play" (right triangle) button. Answer this question by telling us what kind of graph or shape you saw (one sentence should suffice).

   3. Now it's your turn! Try to insert a new cell and type in the code below. Again, don't worry if you don't understand the code, and answer this question by reporting the output (one sentence should suffice).

      ```python
      t = np.arange(0.0, 1.0, 0.01)
      fig = figure(1)
      ax1 = fig.add_subplot(211)
      ax1.plot(t, sin(2*pi*t))
      ax1.grid(True)
      ax1.set_ylim( (-2,2) )
      ax1.set_ylabel('1 Hz')
      ax1.set_title('What is this magic?')
      ```

   Finally, you need to submit this Virtual Lab with your written homework. Inside your VM's terminal, run

   `ipython nbconvert FILENAME.ipynb --to latex --post PDF --SphinxTransformer.author='YOUR_NAME_HERE'`

   This will create a pdf file named `python_sample.pdf`. Merge this pdf with the pdf containing your solution to the written homework (there are many utilities and online tools to do this, one of which is `http://www.pdfmerge.com/`, and submit **hw1.pdf** as usual. In addition, you need to submit a **hw1.zip** file, which contains your code for this Virtual Lab ("python_sample.ipynb"). We want you to zip your code because later in the course, you may be asked to complete multiple notebooks.

   Congratulations, and we hope you enjoyed learning a lot of new stuff in this first homework! As part of the homework, you should start reviewing Python for the upcoming Virtual Labs (EECS 61A is a prerequisite to this class). The course staff will also post some basic review material on Piazza in a few days.

2. Getting started

What is Anant Sahai's second favorite mathematician?

The answer is found on Piazza.

(Why are we having you do this? Piazza is your best source for recent announcements, clarifications on homeworks, and related matters, and we want you to be familiar with how to read the newsgroup.)

3. Implications: Which are true?
Which of the following statements are true? Briefly explain your answers.

1. If 30 is divisible by 10 then 40 is divisible by 10.
2. If 30 is divisible by 9 then 40 is divisible by 10.
3. If 30 is divisible by 10 then 40 is divisible by 9.
4. If 30 is divisible by 9 then 40 is divisible by 9.

4. Karnaugh Maps

Below is the truth table for the boolean function

$$Y = (\neg A \wedge \neg B \wedge C) \vee (\neg A \wedge B \wedge \neg C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge C).$$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

In this question, we will explore a different way of representing a truth table, the *Karnaugh map*. A Karnaugh map is just a grid-like representation of a truth table, but as we will see, the mode of presentation can give more insight. The values inside the squares are copied from the output column of the truth table, so there is one square in the map for every row in the truth table.

Around the edge of the Karnaugh map are the values of the input variables. Note that the sequence of numbers across the top of the map is not in binary sequence, which would be 00, 01, 10, 11. It is instead 00, 01, 11, 10, which is called *Gray code* sequence. Gray code sequence only changes one binary bit as we go from one number to the next in the sequence. That means that adjacent cells will only vary by one bit, or Boolean variable. In other words, *cells sharing common Boolean variables are adjacent.*
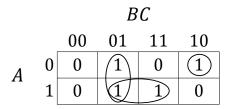
For example, here is the Karnaugh map for Y:

$$BC$$

	00	01	11	10
A 0	0	1	0	1
1	0	1	1	0

The Karnaugh map provides a simple and straight-forward method of minimizing boolean expressions by visual inspection. The technique is to examine the Karnaugh map for any groups of adjacent ones that occur, which can be combined to simplify the expression. Note that "adjacent" here means in the modular sense, so adjacency wraps around the top/bottom and left/right of the Karnaugh map; for example, the top-most cell of a column is adjacent to the bottom-most cell of the column.

For example, the ones in the second column in the Karnaugh map above can be combined because $(\neg A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge C)$ simplifies to $(\neg B \wedge C)$. Applying this technique to the Karnaugh map (illustrated below), we obtain the following simplified expression for Y:

$$Y = (\neg B \wedge C) \vee (A \wedge C) \vee (\neg A \wedge B \wedge \neg C).$$

BC

		00	01	11	10
A	0	0	1	0	1
	1	0	1	1	0

1. Write the truth table for the boolean function

$$Z = (\neg A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (\neg A \wedge \neg B \wedge C \wedge \neg D) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (A \wedge \neg B \wedge C \wedge \neg D).$$

2. Using your truth table from Part 1, fill in the Karnaugh map for Z below.

CD

		00	01	11	10
AB	00				
	01				
	11				
	10				

3. Using your Karnaugh map from Part 2, write down a simplified expression for Z.
4. Show that this simplification could also be found algebraically by factoring the expression for Z in (1).

5. A few proofs

Prove or disprove each of the following statements. For each proof, state which of the proof types (as discussed in the Lecture Notes) you used.

1. For all natural numbers n, if n^2 is even then n^5 is even.
2. For all natural numbers n, $n^2 - n + 3$ is odd.
3. For all real numbers x, y, if $x + y \geq 20$ then $x \geq 10$ or $y \geq 10$.
4. For all real numbers r, if r is irrational then r^2 is irrational.

6. Social Network

Suppose that $p_1, p_2, \ldots, p_n$ denote n people where every two people are either friends or strangers. Let Friends(x,y) be the predicate "x and y are friends". Prove or provide a counterexample for the following statements.

1. For all cases with $n = 5$ people, there exists a group of 3 people that are either all friends or all strangers. In mathematical notation we write this as: $\exists (i,j,k) \in \{1,2,\ldots,5\}^3$ such that $i < j < k$ and (Friends$(p_i,p_j) \land$ Friends$(p_j,p_k) \land$ Friends$(p_i,p_k)) \lor (\neg$Friends$(p_i,p_j) \land \neg$Friends$(p_j,p_k) \land \neg$Friends$(p_i,p_k))$.

2. For all cases with $n = 6$ people, there exists a group of 3 people that are either all friends or all strangers. In mathematical notation we write this as: $\exists (i,j,k) \in \{1,2,\ldots,6\}^3$ such that $i < j < k$ and (Friends$(p_i,p_j) \land$ Friends$(p_j,p_k) \land$ Friends$(p_i,p_k)) \lor (\neg$Friends$(p_i,p_j) \land \neg$Friends$(p_j,p_k) \land \neg$Friends$(p_i,p_k))$.

7. A Weighty Proof

You have 10 bags, each containing 100 coins. Nine of the 10 bags contain genuine gold coins, whereas one bag contains fake coins that are visually indistinguishable from the real gold coins. You don't know which bag has the fake coins, but you do know that real gold coins weigh 10g each while fake ones weigh 10.001g each. You can open the bags, look inside them, take out a few coins, mix them up, etc. You have a weighing machine that you can use *exactly once* – on which you can place a bunch of coins, press a button, and obtain a printed slip showing the weight of the coins placed, down to the milligram.

1. Outline a method to determine which bag has the fake coins.

2. Provide a rigorous proof that your method will indeed always identify the bag with the fake coins, while using the weighing machine exactly once.

8. Inductions

Prove the following using induction:

1. For all natural numbers $n > 2$, $2^n > 2n + 1$.

2. For all natural numbers n, $1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2 - 1)$.

3. For all natural numbers n, $\frac{5}{4}8^n + 3^{3n-1}$ is divisible by 19.

9. Write Your Own Problem

Write your own problem related to this week's material and solve it. You may still work in groups to brainstorm problems, but each student should submit a unique problem. What is the problem? How to formulate it? How to solve it? What is the solution?

10. Registering your EECS 70 instructional account

Answering this question is mandatory to stay in the class. It is also worth a free 4 points.

- To answer this question, log in using your instructional account (cs70-???). Instructional account forms will be handed out in the first discussion sections.

- Register your account using complete and correct information. If for some reason you do not have a student ID number, use your birthday (MM/DD/YYYY) and email Katie at katie.the.headta@gmail.com to notify her.

- Double-check your registration by typing "check-register" and pressing enter.

- Verify that all of the information is correct. Correct any mistakes you find.

- "Answer" this question by writing "I have verified that the registration information for my EECS 70 instructional account cs70-??? is complete and correct" (fill in the ??? with your information) and signing your name if possible.

If your information is incorrect, we will not be able to give you any credit for any of your homework. If you do not know how to do any of the steps above, feel free to get as much help as you need from any source (we suggest Piazza, your classmates, or the readers).