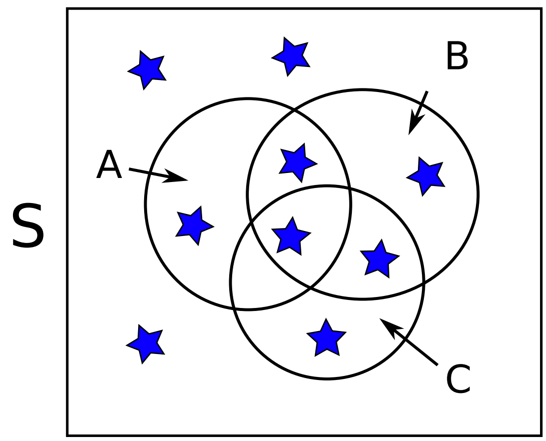
1. The Principle of Inclusion-Exclusion. The Principle of Inclusion-Exclusion states that for events $A_1, \ldots, A_n$ in probability space S,

$$\Pr[\cup_{i=1}^{n} A_i] = \sum_{i=1}^{n} \Pr[A_i] - \sum_{\{i,j\}} \Pr[A_i \cap A_j] + \sum_{\{i,j,k\}} \Pr[A_i \cap A_j \cap A_k] - \cdots + (-1)^{n-1} \Pr[\cap_{i=1}^{n} A_i].$$

That is, the probability of the union of events is the sum of the probabilities, minus the sum of the pairwise intersections (which were counted twice), plus the sum of the 3-way intersections (which were subtracted with the pairwise intersections), etc.

Consider the picture below–this depicts a probability space S, where each of the small stars is one outcome.



(a) How many outcomes are there in S in total? (Hint: Each outcome is represented as a star–just count the number of stars.)

(b) How many outcomes are in the union $A \cup B \cup C$?

(c) What is $\Pr[A \cup B \cup C]$? (Hint: Review the definition of probability in a discrete space.)

(d) Now, we will calculate this probability using the Principle of Inclusion-Exclusion. First, what is $\Pr[A]+\Pr[B]+\Pr[C]$? (Hint: How many outcomes fall in each of the events A,B,C? Take a sum of each of the individual probabilities.)

(e) Notice that the above was a severe overestimate of $\Pr[A\cup B\cup C]$. This is because all of the two-way intersections were counted twice, once for each individual event.

(f) Now, subtract the two-way intersections from the sum of the individual probabilities. What is $\Pr[A]+\Pr[B]+\Pr[C]-(\Pr[A\cap B]+\Pr[B\cap C]+\Pr[C\cap A])$?. (Hint: Simply subtract your answer from part (e) from your answer in part (d), and ensure that your input is of the proper form.)

(g) Finally, notice that this probability is slightly less than $\Pr[A\cup B\cup C]$. This is because the 3-way intersection was originally added 3 times, but then subtracted 3 times, which is once too many.
What is $\Pr[A\cap B\cap C]$?

(h) Now, add back $\Pr[A\cap B\cap C]$ to your answer in the part (f). Does this probability match the probability you calculated in part (c)?