EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai

Discussion 12M-S

1. Random Variables

Consider the following game: you roll two standard 6-sided dice, one after the other. If the number on the first dice divides the number on the second dice, you get 1 point. You get 1 additional point for each prime number you roll.

Define the random variable R_1 to be the result of the first roll, and define R_2 to be the result of the second roll. Define the random variable $X = R_1 + R_2$ to be the sum of the numbers that come up on both dice, define the random variable $Y = R_1 \cdot R_2$ to be the product of the numbers that come up on both dice, and define the random variable Z to be the number of points you win in the game.

(a) How many values can the random variable Z take on (with nonzero probability)?

(b) What are the minimum and maximum values that the random variable Z take on (with nonzero probability)?

(c) Say that your first roll is a 3 and your second roll is a 6. What is the value of Z?

(d) Say that your first roll is a 2 and your second roll is a 1. What is the value of Z?

(e) Say that your first roll is a 4 and your second roll is a 1. What is the value of $X^2 + Y$?

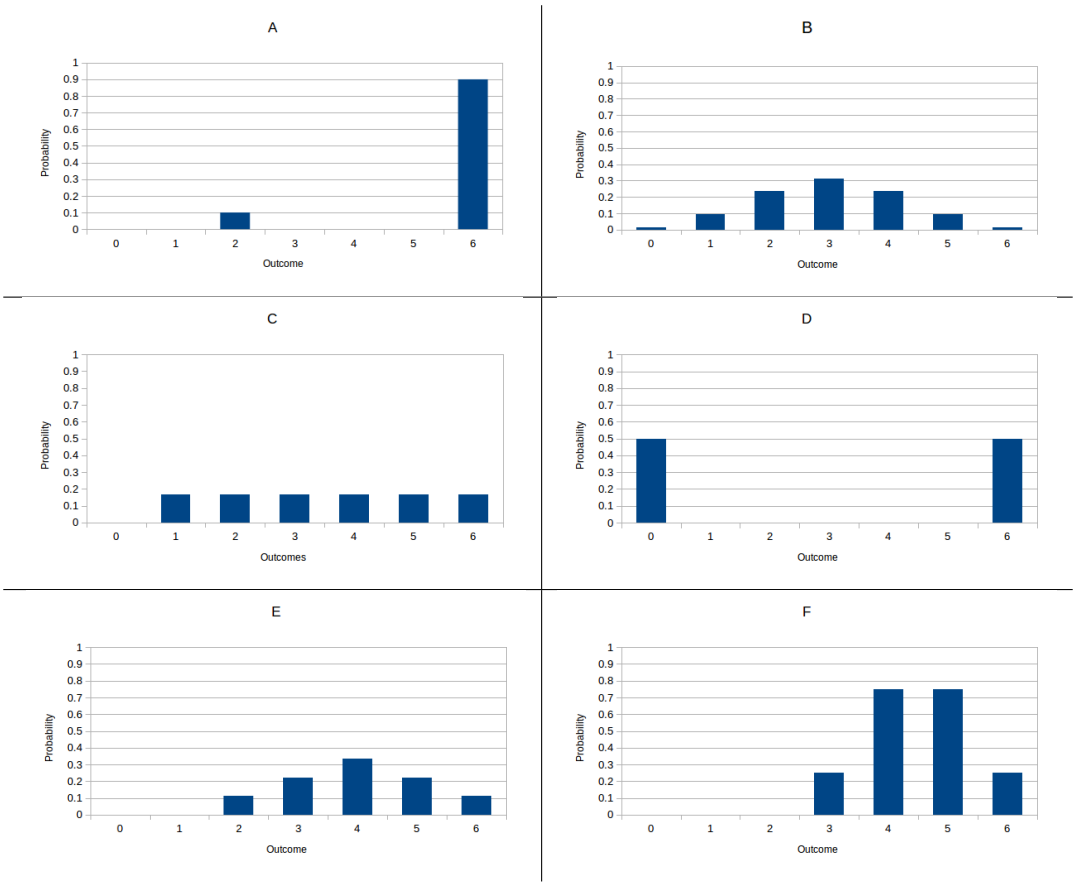
(f) Say that your first roll is a 3 and your second roll is a 5. What is the value of $X + Y + 2 \cdot Z$?

(g) Conditioned on the fact that your second roll is a 1, what is the probability that $Z = 1$?

(h) Conditioned on the fact that your second roll is a 1, what is the probability that $Z = 2$?

2. Distributions of Random Variables

Match each of the 5 random variables below with the correct probability distribution (of the following choices):



(a) What is the distribution corresponding to the number of tails, X, generated in 6 coinflips?

(b) What is the distribution corresponding to the outcome X of rolling a standard 6-sided dice?

(c) What is the distribution corresponding to the sum $X_1 + X_2$ of the outcomes of two 3-sided dice, each with sides labeled 1,2,3?

(d) What is the distribution corresponding to the sum $Z_1 + Z_2 + Z_3$, where Z_i is generated by flipping a coin and setting $Z_i = 2$ if it turns up heads and $Z_i = 1$ if it turns up tails (or, if you like, the sum of three 2-sided dice)?

(e) Say a candy bar is sold for \$6 at the corner store, but is sold for \$2 at the MegaMart one mile away. Because the MegaMart is 9 times further away than the corner store, you are 9 times more likely to buy the candy bar at the corner store. What is the distribution corresponding to the price X you pay for the candy bar on a random day (assuming you choose randomly whether to go to the corner store or the MegaMart with probability proportional to the distance).

(f) When it rains it pours–say that this past April, on half of the days of April there were 0 inches of rain, and the other half of the days there were 6 inches of rain. What is the distribution of X, the number of inches of rain on a uniformly random day of April?

3. A Preview of Expectations

Consider a random variable X which takes on values $x_1, \ldots, x_n$. The *expectation* of X, denoted $\mathbf{E}[X]$, is defined to be

$$\mathbf{E}[X] = \sum_{i=1}^{n} x_i \cdot \mathbb{P}[X = x_i].$$

Notice that when $\mathbb{P}[X = x_i] = \frac{1}{n}$ for all i, then this is simply the familiar notion of an average!

Let us return to the game from the first question: roll two 6-sided dice, award 1 point if the number on the first dice divides the number on the second dice, plus one more point for each prime. Define R_1 to be the result of the first roll, define R_2 to be the result of the second roll, define $X = R_1 + R_2$ to be the sum of the numbers that come up on both dice, define $Y = R_1 \cdot R_2$ to be the product of the numbers that come up on both dice, and define Z to be the number of points you win in the game.

(a) What is $\mathbf{E}[R_1]$?

(b) What is $\mathbf{E}[X] = \mathbf{E}[R_1 + R_2]$?

(c) What is $\mathbf{E}[2 \cdot R_1]$? What do you notice about this expectation?

(d) What is $\mathbf{E}[Z|R_2 = 1]$, the expected number of points we win conditioned on the fact that the second dice roll is a 1? Please enter your answer as a completely reduced fraction (i.e. in the form x/y where x, y are the smallest possible positive integers).