EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai

Discussion 12W-S

1. Expectation and Variance

This problem will give you some practice calculating expectations and variances of random variables.

(a) Suppose that the random variable X takes on 3 values, $10, 25, 70$. Suppose $\mathbb{P}[X = 10] = 0.5$, $\mathbb{P}[X = 25] = 0.2$, and $\mathbb{P}[X = 70] = 0.3$.

i) What is $\mathbf{E}[X]$?

ii) What is $\mathbf{E}[X^2]$?

iii) What is $\text{Var}(X)$?

(b) Let X, Y be random variables, let a, b, c be constants, and let $\mathbf{E}[X] = x$ and $\mathbf{E}[Y] = y$. What is $\mathbf{E}[aX + bY + c]$?

2. A Game–Part I

This game will give you practice with some expectations.

(a) Suppose I have a bag full of equal numbers of \$1 bills and \$5 bills. If I let you choose a bill uniformly at random from this bag, let X be the random variable corresponding to your profit. What is your expected profit?

(b) Say that I now add a second bag, which has equal numbers of \$10 and \$20 bills. Let the random variable corresponding to your profit from this bag be Y. If I let you choose one bill from the first bag and one bill from the second bag, what is your expected proft?

(c) Now, suppose I decide to charge you \$2.50 to draw a bill from the first bag, and \$15.50 to draw from the second bag. If you draw from both bags, what is your expected net profit?

(d) Given the pay scheme above (\$2.50 to draw from the first bag, \$15.50 to draw from the second bag) Which of the following actions gives the maximum expected profit?

i) Drawing from the first bag only.
ii) Drawing from the second bag only.
iii) Drawing from both bags.
iv) Drawing from neither bag.

(e) As the designer of this game, I want your expected profit to be net negative. I have told you that the proportions of \$1 and \$5 bills in the bag are the same. However, I could lie to you and rig the game so that if I charge you \$2.50, your expected profit for drawing from the first bag is negative. What percentage of bills in the bag should be \$1 bils in order to make your expected profit $-\$1$ (round up to the nearest percent)?