EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai **Discussion 11M**

1. **Binomial theorem**

   (a) Consider the expansion of $(x+y)^n$. What is the coefficient of $x^k y^{n-k}$ term? (Assume $0 \le k \le n$.)

   (b) Consider the expansion of $(x_1 + x_2 + \cdots + x_k)^n$. What is the coefficient of $x_1^{c_1} x_2^{c_2} \cdots x_k^{c_k}$ term? ($0 \le c_i \le n$ and $\sum_{i=1}^{k} c_i = n$).

   (c) Show that
$$\sum_{k=0}^{n} \binom{2n}{2k} = \sum_{k=0}^{n-1} \binom{2n}{2k+1}.$$

   (d) Show that
$$\sum_{i=0}^{n} \binom{n}{i} 2^{n-i} = 3^n.$$

2. **Balls in Bins: Independent?** You have k balls and n bins labelled $1, 2, \ldots, n$, where $n \geq 2$. You drop each ball uniformly at random into the bins.

(a) What is the probability that bin n is empty?

(b) What is the probability that bin 1 is non-empty? Argue this both by counting, and by independence.

(c) What is the probability that both bin 1 and bin n are empty?

(d) What is the probability that bin 1 is non-empty and bin n is empty?

(e) What is the probability that bin 1 is non-empty given that bin n is empty?

3. Birthdays

Suppose you record the birthdays of a large group of people, one at a time until you have found a match, i.e., a birthday that has already been recorded. (Assume there are 365 days in a year.)

(a) What is the probability that it takes more than 20 people for this to occur?

(b) What is the probability that it takes exactly 20 people for this to occur?

(c) Suppose instead that you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?