EECS 70 Discrete Mathematics and Probability Theory Fall 2014 Anant Sahai Discussion 12M

1. Let's give you cookies!

The GSIs want to distribute *k* cookies and *k* glasses of milk to students. They will blindly pick a student and give her/him a cookie. Same goes for the glasses of milk.

If there are n students, what should k be such that there is probability at least p every single student gets at least one cookie *and* a glass of milk? What if we have three categories of items: cookies, glasses of milk, and napkins?

2. Locked out

You just rented a large house and the realtor gave you five keys, one for the front door and the other four for each of the four side and back doors of the house. Unfortunately, all keys look identical, so to open the front door, you are forced to try them at random.

Find the distribution and the expectation of the number of trials you will need to open the front door. (Assume that you can mark a key after you've tried opening the front door with it and it doesn't work.)

3. A roll of the dice

Consider a single roll of two dice, one red and one blue.

- (a) Let *R* be the value of the red die. What is the distribution of *R*? What is the expectation of *R*?
- (b) Let *M* be the maximum of the numbers on the two dice. What is the distribution of *M*? What is the expectation of *M*?

(c) How do the distribution and expectation of M compare to that of R?

4. To pay or not to pay?

Alice goes to Berkeley and she drives to school everyday. Tired of always paying for parking, Alice decides one day not to pay her parking fees. Assume that there is a probability of 0.05 that she gets caught by the meter maid. The parking fee is \$0.25 and if she is caught, her parking ticket is \$10.

- (a) How does the expected cost of parking 10 times without paying the meter compare with the cost of paying the meter each time? (*Hint*: Think of Alice getting caught or not as a single biased coin flip with probability 0.05.)
- (b) If she parks at the meter 10 times, what is the probability that she will have to pay more than the total amount he could end up saving by not putting the money?