EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai

Discussion 2W-S

1. Prime Factors

Prove that every integer $n \geq 2$ can be written as a product of prime numbers.

2. Bit String

Prove that every positive integer n can be written with a string of 0s and 1s. In other words, prove that we can write

$$n = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} + \ldots + c_1 \cdot 2^1 + c_0 \cdot 2^0,$$

where $k \in \mathbb{N}$ and $c_k \in \{0,1\}$.

3. Grid Induction

A bug is walking on an infinite 2D grid. He starts at some location $(i, j) \in \mathbb{N}^2$ in the first quadrant, and is constrained to stay in the first quadrant (say, by walls along the x and y axes). Every second he does one of the following (if possible):

(i) Jump one inch down, to $(i, j-1)$.

(ii) Jump one inch left, to $(i-1, j)$.

For example, if he is at $(5,0)$, his only option is to jump left to $(4,0)$.

Prove that no matter how he jumps, he will always reach $(0,0)$ in finite time.