1. Playing Matchmaker

Consider the following list of preferences:

Men	Preferences
A	$4 > 2 > 1 > 3$
B	$2 > 4 > 3 > 1$
C	$4 > 3 > 1 > 2$
D	$3 > 1 > 4 > 2$

Women	Preferences
1	$A > D > B > C$
2	$D > C > A > B$
3	$C > D > B > A$
4	$B > C > A > D$

(a) Is $\{(A,4),(B,2),(C,1),(D,3)\}$ a stable pairing?

(b) Find a stable matching by running the Traditional Propose & Reject algorithm.

(c) Show that there exist a stable matching where all the women get their first choice.

2. Examples or It's Impossible

Determine if each of the situations below is possible with the traditional propose-and-reject algorithm. If so, give an example of size $n \geq 3$. Otherwise, explain briefly why you think it's impossible.

(a) Every man gets his first choice.

(b) Every woman gets her first choice, even though her first choice does not prefer her the most.

(c) Every woman gets her last choice.

(d) Every man gets his last choice.

(e) A man who is second on every woman's list gets his last choice.

3. Universal Preference

Suppose that preferences in a stable marriage instance are universal: all n men share the preferences $W_1 > W_2 > \cdots > W_n$ and all women share the preferences $M_1 > M_2 > \cdots > M_n$.

(a) What result do we get from running the algorithm with men proposing? Can you prove it?

(b) What result do we get from running the algorithm with women proposing?

(c) What does this tell us about the number of stable matchings?