EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai

Discussion 3M-S

1. **Stable Matching** Say that we want to pair up the men (1,2,3) with the women (A,B,C); such a pairing is called a *matching*. The table below gives the ranked preferences of the people, with most preferred on the left and least preferred on the right.

man	ranking
1	B, A, C
2	A, B, C
3	B, A, C

woman	ranking
A	3, 2, 1
B	1, 2, 3
C	2, 1, 3

Given a matching, we say that two people form a *rogue couple* if both prefer each other to the partner they are currently matched to.

a) What are all of the rogue couples for the matching (1A, 2B, 3C) with respect to the preferences in the table above?

b) Again given the preferences above, what are all of the rogue couples for the matching (3A, 2B, 1C)?

2. **Propose-and-Reject Algorithm** A matching is considered *stable* if it contains no rogue couples. This is because no one can convince anyone else to leave their partner in order to form a new pair. The stable marriage algorithm allows us to find stable matchings. This algorithm is described in course note 4. The following problems are designed to give you practice with this algorithm.

Run the stable marriage algorithm for the following ranked preferences between three men/suitors (1,2,3) and three women/suitees (A,B,C).

man	ranking
1	B, A, C
2	A, B, C
3	B, A, C

woman	ranking
A	3, 2, 1
B	1, 2, 3
C	2, 1, 3

a) What is the status of each man after the first round of the algorithm?
Which woman did man 1 make an offer to?
Was he rejected?
Which woman did man 2 make an offer to?
Was he rejected?
Which woman did man 3 make an offer to?
Was he rejected?

b) What is the status of each man after the second round of the algorithm?
Which woman did man 1 make an offer to or remain with at the start of the round?

Was he rejected or kicked out?
Which woman did man 2 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?
Which woman did man 3 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?

c) What is the status of each man after the third round of the algorithm?
Which woman did man 1 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?
Which woman did man 2 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?
Which woman did man 3 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?

d)What is the status of each man after the fourth round of the algorithm?
Which woman did man 1 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?
Which woman did man 2 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?
Which woman did man 3 make an offer to or remain with at the start of the round?
Was he rejected or kicked out?