1. Beginner Chinese Remainder Theorem

Solve for $x \in \mathbb{Z}$ where

$$x \equiv 3 \pmod{11}$$
$$x \equiv 7 \pmod{13}$$

(a) Find the multiplicative inverse of 13 modulo 11.

(b) What is the smallest $b \in \mathbb{Z}^+$ such that $13 \mid b$ and $b \equiv 3 \pmod{11}$?

(c) Find the multiplicative inverse of 11 modulo 13.

(d) What is the smallest $a \in \mathbb{Z}^+$ such that $11 \mid a$ and $a \equiv 7 \pmod{13}$?

(e) Now, write a set of possible x

2. More Chinese Remainder Theorem

Solve for $x \in \mathbb{Z}$ where

$$x \equiv 2 \bmod 3$$
$$x \equiv 3 \bmod 5$$
$$x \equiv 4 \bmod 7$$

(a) Find the multiplicative inverse of 5×7 modulo 3.

(b) What is the smallest $a \in \mathbb{Z}^+$ such that $5 \mid a, 7 \mid a$ and $a \equiv 2 (\bmod 3)$?

(c) Find the multiplicative inverse of 3×7 modulo 5.

(d) What is the smallest $b \in \mathbb{Z}^+$ such that $3 \mid b, 7 \mid b$ and $b \equiv 3 (\bmod 5)$?

(e) Find the multiplicative inverse of 3×5 modulo 7.

(f) What is the smallest $c \in \mathbb{Z}^+$ such that $3 \mid c, 5 \mid c$ and $c \equiv 4 (\bmod 7)$?

(g) Write a set of possible x.

3. **Discussion Troubles** One day in EECS70 discussion, the GSI tells students to work in groups. For the first problem, s/he tells everyone to work in pairs, but there is one student left out. The next problem, they work in groups of 4, but again there is one student left out. Again when they work in groups of 10, one is left out. But when they work in groups of 6, three students are left out. The GSI knows that there are less than 30 students in the class. How many students are in the class? (Also, what is the smallest size of groups s/he can split students into, such that nobody is left out?)