EECS 70 Discrete Mathematics and Probability Theory
Fall 2014 Anant Sahai

Discussion 7W

1. Erasure warm-up

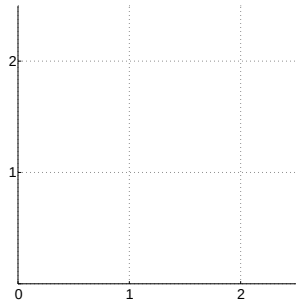
Working over $GF(q)$, you want to send your friend a message of $n = 4$ packets and guard against 2 lost packets. What is the minimum q you can use? What is the maximum degree of the unique polynomial that describes your message?

2. Visualizing error correction

Alice wants to send a message of 2 packets to Bob, and wants to guard against 1 lost packet. So working over $GF(3)$, she finds the unique polynomial $P(x)$ that passes through the points she wants to send, and sends Bob her augmented message of 3 packets: $(0,P(0)),(1,P(1)),(2,P(2))$.

One packet is lost, so Bob receives the following packets: $(0,2),(2,0)$.

(a) Plot the points represented by the packets Bob received on the grid below.



(b) Draw in the unique polynomial $P(x)$ that connects these two points.

(c) By visual inspection, find the lost packet $(1,P(1))$.

3. Where are my packets?

Alice wants to send the message (a_0,a_1,a_2) to Bob, where each $a_i \in \{0,1,2,3,4\}$. She encodes it as a polynomial P of degree ≤ 2 over $GF(5)$ such that $P(0) = a_0$, $P(1) = a_1$, and $P(2) = a_2$, and she sends the packets $(0,P(0))$, $(1,P(1))$, $(2,P(2))$, $(3,P(3))$, $(4,P(4))$. Two packets are dropped, and Bob only learns that $P(0) = 4$, $P(3) = 1$, and $P(4) = 2$. Help Bob recover Alice's message.

(a) Find the multiplicative inverses of $1,2,3$ and 4 modulo 5.

(b) Find the original polynomial P by using Lagrange interpolation or by solving a system of linear equations.

(c) Recover Alice's original message.

4. More erasures!

Consider the alphabet $A = 0$, $B = 1$, $C = 2$, $D = 3$, $E = 4$. Suppose a message of length 3 is sent using the error correction scheme discussed in class over $GF(5)$. If you receive the following packets, what was the original message?

(a) C _ A A

(b) _ A C C

(c) Can you determine the original message if you only receive C E _ _? Either find the original message or explain why you can't.

5. Polynomial divisibility

Let $A(x)$, $B(x)$, $Q(x)$, and $R(x)$ be 4 polynomials, where $A(x) = B(x)Q(x) + R(x)$, with $0 \leq \deg R(x) < \deg B(x) \leq \deg A(x)$.

Prove that a polynomial $C(x)$ divides $A(x)$ and $B(x)$ if and only if it divides $B(x)$ and $R(x)$.