1. **Probability Practice**

   (a) A message source *M* of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet $\{0,1,2\}$, and all such words are equally probable. What is the probability that *M* produces a word that looks like a byte (*i.e.*, no appearance of '2')?

   (b) If five numbers are selected at random from the set $\{1,2,3,\ldots,20\}$, what is the probability that their minimum is larger than 5? (A number can be chosen more than once.)

2. **Clinical tests**

   You may want to use a calculator for this problem.

   Let's say that there is a rare disease, and only $p = 1\%$ of the human population has that disease.

   Let's also assume that there is a clinical test available for the disease, but the test is not perfect. The chances that the test is accurate are only $q = 90\%$. So, if 1000 people who have the disease are tested, the test will come out positive in only about 900 of these cases. Likewise, if 1000 healthy individuals are tested, the test will come back negative in only about 900 of these cases.

   (a) Suppose an individual drawn randomly from this population tests positive. What are the chances that this individual actually has the disease?

(b) Suppose an individual drawn randomly from this population tests negative. What are the chances that this individual does not actually have the disease?

(c) Suppose a group of scientists get together and develop a new test that has an accuracy of $q = 99.9\%$ (a major improvement that replaces the old test overnight). Now how do the chances in parts (a) and (b) above change?

3. **Best choice problem**

Three princes are going to stop by Alice's house and invite her to their parties. Alice has not met them before so she does not know who she likes most. There are two strategies:

  1 Go with the first prince.

  2 Reject the first prince. If she find she likes the second prince more than the first one, she will go with him. Otherwise, she will go with the third prince.

Which strategy gives Alice the highest probability to go with the prince she likes the most?